Testing for the implicit weights of the dimensions of the Human Development Index using stochastic dominance

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Abstract

In 2010, United Nations' Development Programme changed the indicators used to obtain education and income indices in the Human Development Index (HDI). In this paper, we use the Stochastic Dominance Efficiency methodology to evaluate the implicit weights of the dimensions used in the new measurement of the HDI. We find, contrary to the earlier literature, that the implicit weight attached to the education dimension is relatively low suggesting that it is relatively harder to achieve high scores in this dimension compared to other dimensions.

JEL Classifications: C14; I31; O15

Key Words: well-being; human development; composite indices; nonparametric stochastic dominance

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1. Introduction

The most well-known and accepted composite index measuring the composite well-being of countries is the United Nations' Human Development Index (HDI), which is a geometric mean of the education, life expectancy and GNI per capita indices, where these dimensions are assigned equal weights to reflect equal intrinsic value given to each dimension (see e.g., Alkire and Santos, 2014). However, each indicator is normalized through a transformation which leads to weights that are implicitly different than the explicit ones. Normalization of two components might suggest that one component is given implicitly more weight than another one if the difference between the upper and lower bound is relatively low for one component and relatively high for the other one. Then the effect of the former on the composite index becomes somewhat higher than that of the latter (Noorbakhsh, 1998). For instance, one could still keep the explicit weight of a dimension to be the same, but could decrease the upper goalpost for this dimension (e.g., a country can be assigned a full normalized score if it achieves an average of 80 years of life expectancy rather than 85 years of life expectancy in the health dimension). This would be equivalent to increasing the implicit weight of this dimension to arrive at higher levels of human development without any change in the pre-assigned explicit weights. As a result, even though each sub-index is assigned an explicit equal weight, after transforming the raw components into an index, each sub-index gets assigned a different implicit weight. Clearly, the upper and lower bounds used to transform the indicators to obtain the sub-indices of the HDI are somehow arbitrary and will impose different implicit weights to dimensions.

In this paper, we will adopt a data driven alternative weighting scheme to arrive at a composite index based on stochastic dominance efficiency (SDE) analysis that will shed a different light on the implicit weights assigned to dimensions of the HDI. The weights derived from SDE analysis can be thought of as weights that lead to the most optimistic well-being scenario and hence identify whether pre-determined weights impose higher (or lower) implicit weights to some dimensions if one were to measure relative welfare across countries. Since each sub-index in the HDI is bounded between 0 and 1, higher measured human development levels for more countries describe a distribution that is negatively skewed resulting in less variability across countries. As such, SDE analysis applied to scaled data would result in the most optimistic composite index in which more observations correspond to higher measured relative development levels.

In a recent paper, Pinar et al. (2013) used the SDE methodology to obtain the best-case scenario weighting scheme for the sub-indices of the HDI used prior to 2010 edition.[1] They found that weighting the education index relatively more than the pre-determined equal weights would lead to a more optimistic measurement of welfare. Since most countries have already achieved good levels of literacy and enrolment ratios (i.e., indicators considered to measure the education index prior to the 2010 edition of Human Development Report), it has been suggested that these indicators do not serve a purpose any longer for relative welfare comparisons since attaching an equal weight to the education index would allow most countries to achieve one third of so-called human development. The United Nations' Development Programme changed the way the education dimension measured after their 2010 edition to address this issue. Here, we follow the same methodology to test whether the new measurement of the HDI results in implicit

[1]The transformation of raw components into an index until 2010 is defined as follows. The value of a country's life expectancy index is obtained by the country's life expectancy in years minus 25 divided by 60, for a number that would lie between 0 and 1. The education index (EI) is defined as EI= 2/3 (adult literacy index) + 1/3 (gross enrolment index). This index is constructed so that a 2/3 weight is given to literacy (percentage of the population that is considered literate) and a 1/3 weight is given to gross school enrolment as a percentage of the eligible school age population and it is bounded between 0 and 1. The GDP per capita index is defined as, GDP Index =

$$\frac{\log(\text{GDP per capita}) - \log(100)}{\log(40000) - \log(100)}.$$

weights that are similar to the explicit ones and examine which dimensions receive relatively higher (lower) implicit weights.

From a policy point of view, the reason behind using equal explicit weights is to ensure that each dimension receives the same importance. However, this would not be the case in practice if it is relatively less costly to achieve higher “human development” outcomes by allocating resources in certain dimensions than others. In other words, if the implicit weights are different than the explicit ones, the construction of the HDI could reveal policy incentives for countries to better allocate their resources to improve their “human development” outcomes. The methodology used in this paper will result in obtaining the implicit dimension weights by which countries have increased their measured “human development” over-time. This is not to say that the component that contributes more to the improvement of the index is the most efficient one (since this would depend on the costs of improving each particular dimension of the HDI). However this method would highlight the dimensions that are implicitly favoured (given more importance) by most countries, something that would allow policymakers to obtain a method equating implicit and explicit weights so that the intended message (i.e., assigning equal importance to each dimension) can be implemented in practice.

In the next section we discuss the new measurement of the dimensions used to construct the HDI. Next we present the main framework of our analysis using SDE. Finally, we present the data, findings of the empirical application, and a discussion in light of the empirical results.

2. New measurement of dimensions in the HDI

We use the United Nations’ Development Program’s HDI and its sub-indices - health, education, and income indices for 1990, 1995, 2005, 2010, 2011, 2012, 2013, 2014, and 2015. HDI scores are obtained as the geometric average of the three sub-indices, where each index is obtained through a normalization procedure by setting minimum and maximum (goalposts) in order to set the index values between 0 and 1.[2]

The health sub-index is measured by life expectancy (LE) at birth component, and normalized sub-index outcomes are obtained by using minimum and maximum goalposts of 20 and 85 years of life expectancy, respectively. Hence, the health index (HI) outcome of a given country is obtained by using the following normalization procedure $HI = \frac{LE-20}{85-20}$ where LE is the life expectancy of a given country.

The education sub-index is measured by mean of years of schooling (MYS) for adults aged 25 years or above and expected years of schooling (EYS) for children of school entering age. Index values for MYS and EYS (MYSI and EYSI, respectively) are obtained by using a minimum values of zero and maximum values of 15 and 18 years respectively, $MYSI = \frac{MYS-0}{15-0}$ and $EYSI = \frac{EYS-0}{18-0}$, respectively. Then, two indices are combined into an education index (EI) using arithmetic mean, $EI = \frac{MYSI+EYSI}{2}$.

Finally, income index (II) is calculated by using the normalization procedure $II = \frac{\ln(\text{GNI per capita}) - \ln(100)}{\ln(75000) - \ln(100)}$ where the minimum and maximum goalposts for gross national income (GNI) per capita are \$100 and \$75,000, respectively.

The indicators used to obtain the education and income indices were different before the 2010 HDI edition. The income dimension in the previous measurement used gross domestic product (GDP) per capita and the maximum goalpost for this dimension was \$40,000. On the

[2]For details, please refer to Human Development Report 2016 technical notes: http://dev-hdr.pantheonsite.io/sites/default/files/hdr2016_technical_notes_0.pdf

other hand, the education dimension used literacy rates (percentage of population that is considered as literate) and a gross school enrolment ratio (percentage of students enrolled of the eligible school age population). The upper goalpost for the income dimension is now relatively higher and the indicators used to obtain the education dimension are relatively harder to achieve when compared to its previous indicators used to construct this dimension.

The descriptive statistics for the sub-components of HDI suggest that the health index has the highest mean (median) over-time followed by the income and education indices, respectively (see the Online Appendix Table A.1 for the details). Furthermore, the health index displays noticeably more negative skewness than the other sub-indices suggesting that the majority of the observations are clustered at the upper tail of the distribution.

3. Stochastic Dominance Efficiency

This section discusses briefly the SDE methodology that we employ. We consider a $3 \times N$ matrix of achievements $\mathbf{Y}$ taking values in $\mathbb{R}^3$, where the observations consist of a realization of achievements in three sub-components of the HDI for N number of countries. We denote by $F(\mathbf{y})$, the continuous cumulative distribution function (cdf) of $\mathbf{Y} = (Y_1, Y_2, Y_3)'$ at point $\mathbf{y} = (y_1, y_2, y_3)'$. Let us consider a weighting vector $\boldsymbol{\lambda} \in \mathbb{L}$ where $\mathbb{L} := \{\boldsymbol{\lambda} \in \mathbb{R}_+^3 \ : \ \mathbf{e}'\boldsymbol{\lambda} = 1\}$ with $\mathbf{e}$ being a vector of ones suggesting that all sub-indices have non-negative weights that sum up to one. We denote by $G(z, \boldsymbol{\lambda}; F)$ the cdf of the composite index value for $\boldsymbol{\lambda}'\mathbf{Y}$ at point z given by $G(z, \boldsymbol{\lambda}; F) := \int_{\mathbb{R}^3} \mathbb{I}\{\boldsymbol{\lambda}'\mathbf{u} \le z\} dF(\mathbf{u})$.

Stochastic dominance (SD) is used to test a set of relations that may hold between distributions (e.g., comparison of a distribution of a given well-being dimension across countries). SDE is a direct extension of SD to the case where full diversification is allowed (i.e., comparison of weighted achievement distributions across countries). In the full diversification case, the composite achievement levels of countries with pre-determined weights (e.g., the composite index obtained with a vector of equal weights, $\boldsymbol{\tau}$), are taken as a benchmark and tested against all possible composite indices produced with any possible weighting scheme allocated to the sub-indices. To do this, we use the following objective function:

$$\underset{\lambda}{Max}[G(z,\boldsymbol{\tau};F)-G(z,\boldsymbol{\lambda};F)] \text{ for a given } z$$

The above maximization results in the most optimistic well-being index with $\boldsymbol{\lambda}$ in the sense that it reaches the highest level of measured well-being for a given well-being level, implying that the number of observations having relative well-being levels above a given z value is maximized. The z values are obtained from the equally-weighted index as the number of observations above z is known. The maximization problem above finds $\boldsymbol{\lambda}$ that gives the lowest number of observations above for each z value. The above objective function is based on the first-order SD criterion, a relatively less restrictive criterion in which all types of utility functions are considered as long as they are non-decreasing in well-being.

The general hypotheses for testing the first-order of SDE of the composite well-being index obtained with equally-weighted vector can be written compactly as:

$$\begin{aligned} H_0 &: G(z,\boldsymbol{\tau};F) \leq G(z,\boldsymbol{\lambda};F) \text{ for all } z \in \mathbb{R} \text{ and for all } \boldsymbol{\lambda} \in \mathbb{L}, \\ H_1 &: G(z,\boldsymbol{\tau};F) > G(z,\boldsymbol{\lambda};F) \text{ for some } z \in \mathbb{R} \text{ or for some } \boldsymbol{\lambda} \in \mathbb{L}. \end{aligned}$$

In order to test the null hypothesis, we consider the weighted Kolmogorov-Smirnov type test statistic:

$$\hat{S} := \sqrt{N}\frac{1}{N}\sup_{\lambda}\left[G(z,\boldsymbol{\tau};\hat{F})-G(z,\boldsymbol{\lambda};\hat{F})\right] \text{ for a given } z \text{ level,}$$

and a test based on the decision rule:

$$\text{reject } H_0 \text{ if } \hat{S} > c,$$

where c is some critical value (for the derivation of the test, see Pinar et al., 2013). Since the distribution of the test statistic depends on the underlying distribution, we rely on a subsampling bootstrap method adopted by Linton et al. (2005).

4. Results and discussion

Using the SDE methodology we find that the equal weight allocation to sub-indices does not result in the most optimistic well-being scenario and there are many composite indices constructed with alternative weights λ that stochastically dominate the equally-weighted HDI in the first-order sense.[3] We conducted our analysis for different periods in order to understand the over-time evolution of most optimistic weight allocation to sub-indices. Table 1 presents the over-time most optimistic weighting scheme of the sub-components of HDI where we have a different number of country coverage over-time (the number of countries in each period is reported in the second column). We find that there are many alternative ways of combining the sub-components that would dominate the equally-weighted case and the weights represented in Table 1 are the average of the dominating cases. In all cases, we reject the null hypothesis of non-dominance at the 1% significance level (p-values of the bootstrap tests are provided in the last column of Table 1).

[3]We use the arithmetic average of the HDI components for our analysis as our methodology obtains the most optimistic weight allocation across sub-indices with a linear optimization problem. However, there is a little difference in achievement levels of countries when calculated with the arithmetic or geometric mean (e.g., median difference in composite HDI scores obtained with both aggregation is only 0.006 in 2015) and different ways of calculating the final composite index does not affect the qualitative findings of this paper.

Table 1
Most optimistic weighting scheme of the sub-indices of the HDI

Year	Number of observations	Dominating number of observations	Health index	Education index	Income index	Bootstrap p-values
1990	144	144	0.778	0.000	0.222	0.0000
1995	148	148	0.812	0.000	0.188	0.0000
2000	168	168	0.815	0.006	0.179	0.0083
2005	182	182	0.857	0.033	0.110	0.0094
2010	188	188	0.896	0.017	0.087	0.0090
2011	188	188	0.909	0.007	0.084	0.0087
2012	188	188	0.899	0.000	0.101	0.0089
2013	188	188	0.901	0.000	0.099	0.0083
2014	188	188	0.899	0.000	0.101	0.0075
2015	188	188	0.897	0.000	0.103	0.0060

The health index always contributes more than an equal weight to the most optimistic HDI, a weight allocation that ranges roughly between 78% and 91% over time. On the other hand, the income index is the second main contributor to the most optimistic HDI with the weight ranging roughly between 8% and 22%. Finally, the education index does not contribute to the most of optimistic case for the most of the time and its highest contribution to the most optimistic HDI is roughly 3%.

To show the impact of the weighting scheme on the measured well-being outcomes, Fig. 1 gives the cumulative composite index outcomes with the benchmark HDI (i.e., equally-weighted HDI) and the optimistic HDI (i.e., index outcomes obtained with the most optimistic weighting scheme) in 2015. The vertical and horizontal axes present the composite index outcome and the number of countries, respectively. This figure clearly shows that there is a remarkable improvement in measured outcomes if one were to weight the health index relatively more as there is a clear first-order dominance of the most optimistic case over the benchmark one.

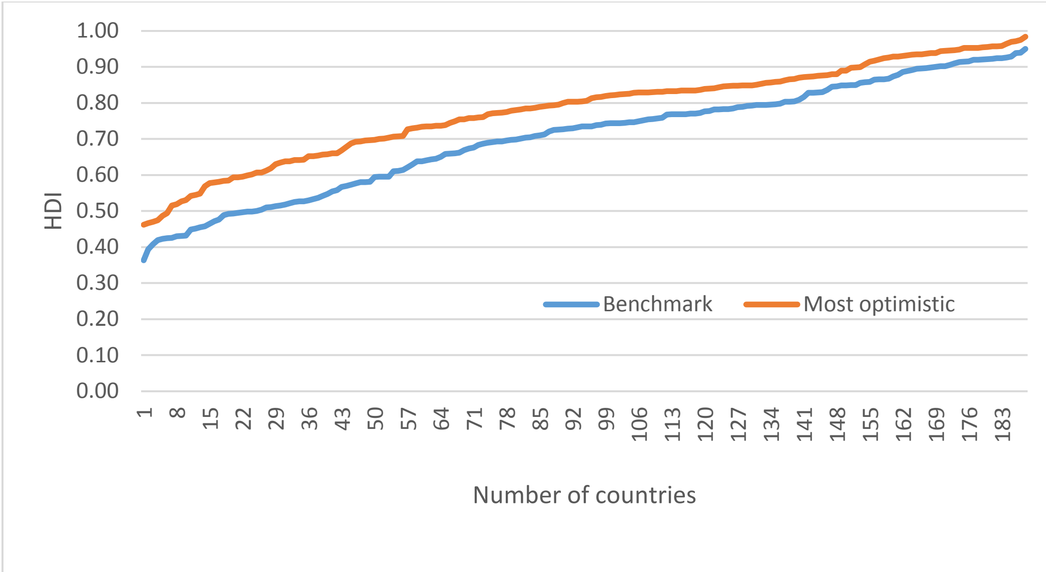

Fig.1. Cumulative distributions of achievements with the benchmark and optimistic HDI

What does the weight allocation across the sub-components of HDI that produces the most optimistic HDI tell us? Clearly, the majority of countries have achieved better health outcomes (i.e., most of the countries experienced relatively high outcomes in this index compared to the upper bounds set in other dimensions). The health index has always had the highest mean (and median) compared to the other sub-components over-time suggesting that the upper bound of this index is relatively more achievable compared to the other indices, hence implicitly receiving relatively higher weight. Given the trade-offs across the three dimensions of the HDI, countries can make strategic decision which would enable them to increase their measured HDI by placing a higher implicit importance to those dimensions identified by the SDE approach as the most effective ones to reach that goal. The weights obtained with the SDE approach shows that health dimension is not only the most achieved dimension by the majority of countries, but it is also the dimension that offers a cost effective way of increasing the measured “human development” compared to the standard of living dimension. Ravallion (2012) calculated income levels that countries need to forego to increase one year of life expectancy while keeping the HDI scores constant. This amount is extremely high for almost all countries compared to the actual marginal cost of increasing a year of life expectancy.[4] In other words, if a country's target is to increase its HDI score, the most cost-effective way of doing this is to invest in its health system, since the marginal cost of doing so is lower compared to the gains that a country can make by allocating this amount to income per capita dimension.

Both the income and education index, on the other hand, contribute less than the equal weights to the most optimistic case. The natural logarithm transformation could be considered as a well-suited transformation for the income index due to the skewness of GNI per capita levels, but the upper bound is relatively harder to achieve when compared to the health index, an issue that requires additional consideration by policymakers. Finally, the education index contributes very little to the most optimistic case suggesting that the achievement levels in this dimension are relatively low when compared with the other two sub-indices. The upper goalposts set for the indicators used for the education index are relatively harder to achieve and hence this dimension receives relatively low implicit weights. Given the relative achievements of countries in the education dimension, policymakers should consider decreasing the upper goalposts in the MYS and EYS. Our findings are in line with the findings of Ravallion (2012) who found that the new HDI's valuations of extra schooling are now very high. The economic return of an additional year of schooling is four times lower than the valuation placed to additional years of schooling in the HDI, which makes investment to education less attractive given the measurement of the HDI. On the other hand, Pinar et al. (2013) found that the implicit weights of the education dimension indicators were relatively high with its previous measurement as the majority of countries achieved relatively better outcomes in these indicators (i.e., literacy rates and enrolment ratios)

[4] See Figure 3 of Ravallion (2012) which reports the implicit valuations in HDI and marginal costs of an extra year of life expectancy.

compared to the other dimensions. Comparing the findings of Pinar et al. (2013) with the use of the new measurement of education used in this paper, we find that the change of indicators in the education index from literacy rates and gross enrolment ratios to MYS and EYS is a good step forward, since the previous measurement imposed a relatively higher implicit weight to this dimension. However, the upper goalposts set for the current measurement of indicators would need to be lowered in order to make this dimension's implicit importance count more in the new measurement of the HDI.

In summary, our results suggest that even though the intention of policymakers is to place equal importance to each sub-index by giving equal weight to each dimension explicitly, this has not been the case in practice. We find that each sub-index receives a different level of implicit weight because of the normalization procedure used for the indicators in each sub-index.

We also conduct a ranking analysis of HDI scores obtained with the optimistic weighting scheme when compared to the geometric mean aggregation in 2015.[5] Rankings obtained with the most optimistic case scenario are distinctively different from the one obtained with the geometric mean and we will briefly describe the main findings below. Countries that are ranked at the top-20 with the optimistic weighting scheme are the ones that have good health outcomes (see Table A.2 of the Online Appendix). For instance, Hong Kong, Singapore, and Japan moved to the top three in the rankings with the optimistic case since these countries not only have good health outcomes but also have high standard of living. Furthermore, other countries that moved to the top-20 with the optimistic one are Mediterranean countries that are famously known for high life expectancy rates. In particular, Italy, France, and Spain moved to the 5^{th}, 8^{th} and 11^{th} positions with the optimistic case scenario compared to the positions held with the geometric mean aggregation (i.e., 26^{th}, 27^{th}, and 21^{st}, respectively).

[5] See the Online Appendix for the detailed rankings analysis.

Some of the countries experienced large movements in the rankings with the optimistic weighting scheme compared to the geometric mean. We looked at the countries that moved to the higher and lower rankings with the optimistic case more than 25 positions compared to rankings obtained with the geometric mean aggregation (see Table A.3 of the Online Appendix). The majority of the Central American countries (Nicaragua, Honduras, Costa Rica, and Mexico), some of the Asian countries (China, Vietnam, Bangladesh) and Caribbean countries (Dominica, Cuba, Jamaica) and a good number of African countries (Morocco, Eritrea, Niger, Ethiopia, and Cape Verde) moved to relatively higher rankings with the optimistic scenario. These countries have relatively higher achievements in the health dimension but their achievements in education and standard of living are relatively lower. On the other hand, the majority of countries that moved to relatively lower rankings with the optimistic weighting scheme are countries that were part of the Soviet Union (Russian Federation, Kazakhstan, Belarus, Lithuania, Azerbaijan, Ukraine, and Latvia) which have good educational systems compared to many countries but have relatively low achievements in the health dimension. A cluster of African countries (South Africa, Swaziland, Botswana, Angola, Nigeria, Equatorial Guinea, Gabon, and Lesotho) also moved to relatively lower rankings since these countries have more or less balanced achievements across all dimensions, something favoured by the geometric mean aggregation.[6]

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[6] See the Table A.4 of the Online Appendix for the full set of rankings obtained for 2015 with the optimistic case scenario and geometric mean aggregation.

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Table A.1
Descriptive Statistics

Year\Index	***Health index***	***Education index***	***Income index***
1990			
Mean	0.702	0.481	0.637
Median	0.745	0.502	0.641
Minimum	0.207	0.081	0.194
Maximum	0.908	0.873	1.000
Skewness	-0.941	-0.322	-0.113
Standard Deviation	0.148	0.180	0.184
Number of countries	144	144	144
Proportion < 0.3	0.014	0.208	0.014
Proportion ≥ 0.3 & ≤ 0.7	0.396	0.729	0.583
Proportion > 0.7	0.590	0.063	0.403
1995			
Mean	0.711	0.517	0.633
Median	0.751	0.549	0.634
Minimum	0.177	0.097	0.191
Maximum	0.922	0.894	1.000
Skewness	-0.973	-0.311	-0.044
Standard Deviation	0.153	0.187	0.188
Number of countries	148	148	148
Proportion < 0.3	0.014	0.182	0.027
Proportion ≥ 0.3 & ≤ 0.7	0.338	0.642	0.595
Proportion > 0.7	0.648	0.176	0.378
2000			
Mean	0.720	0.551	0.642
Median	0.771	0.585	0.649
Minimum	0.288	0.116	0.213
Maximum	0.941	0.895	1.000
Skewness	-0.793	-0.330	-0.058
Standard Deviation	0.158	0.187	0.193
Number of countries	168	168	168
Proportion < 0.3	0.006	0.131	0.030
Proportion ≥ 0.3 & ≤ 0.7	0.339	0.655	0.571
Proportion > 0.7	0.655	0.214	0.399
2005			
Mean	0.741	0.583	0.654
Median	0.792	0.606	0.666
Minimum	0.335	0.146	0.202
Maximum	0.958	0.905	1.000
Skewness	-0.773	-0.288	-0.136
Standard Deviation	0.152	0.183	0.191
Number of countries	182	182	182
Proportion < 0.3	0.000	0.066	0.022
Proportion ≥ 0.3 & ≤ 0.7	0.330	0.632	0.538
Proportion > 0.7	0.670	0.302	0.440
2010			
Mean	0.768	0.616	0.675
Median	0.806	0.640	0.690
Minimum	0.424	0.181	0.263
Maximum	0.970	0.928	1.000
Skewness	-0.706	-0.313	-0.220
Standard Deviation	0.137	0.176	0.182
Number of countries	188	188	188
Proportion < 0.3	0.000	0.053	0.016
Proportion ≥ 0.3 & ≤ 0.7	0.282	0.612	0.505
Proportion > 0.7	0.718	0.335	0.479

2011			
Mean	0.773	0.623	0.677
Median	0.809	0.649	0.693
Minimum	0.436	0.191	0.266
Maximum	0.974	0.932	1.000
Skewness	-0.694	-0.323	-0.233
Standard Deviation	0.134	0.174	0.181
Number of countries	188	188	188
Proportion < 0.3	0.000	0.037	0.016
Proportion ≥ 0.3 & ≤ 0.7	0.282	0.606	0.511
Proportion > 0.7	0.718	0.356	0.473
2012			
Mean	0.778	0.629	0.680
Median	0.813	0.655	0.696
Minimum	0.445	0.199	0.272
Maximum	0.978	0.936	1.000
Skewness	-0.683	-0.349	-0.251
Standard Deviation	0.132	0.173	0.180
Number of countries	188	188	188
Proportion < 0.3	0.000	0.032	0.016
Proportion ≥ 0.3 & ≤ 0.7	0.271	0.585	0.505
Proportion > 0.7	0.729	0.383	0.479
2013			
Mean	0.782	0.634	0.683
Median	0.816	0.658	0.698
Minimum	0.446	0.201	0.263
Maximum	0.981	0.939	1.000
Skewness	-0.676	-0.330	-0.277
Standard Deviation	0.130	0.174	0.179
Number of countries	188	188	188
Proportion < 0.3	0.000	0.027	0.022
Proportion ≥ 0.3 & ≤ 0.7	0.255	0.569	0.489
Proportion > 0.7	0.745	0.404	0.489
2014			
Mean	0.786	0.638	0.685
Median	0.819	0.659	0.702
Minimum	0.446	0.206	0.262
Maximum	0.985	0.939	1.000
Skewness	-0.669	-0.340	-0.279
Standard Deviation	0.129	0.174	0.179
Number of countries	188	188	188
Proportion < 0.3	0.000	0.027	0.016
Proportion ≥ 0.3 & ≤ 0.7	0.250	0.553	0.479
Proportion > 0.7	0.750	0.420	0.505
2015			
Mean	0.790	0.639	0.687
Median	0.822	0.659	0.702
Minimum	0.445	0.206	0.267
Maximum	0.987	0.939	1.000
Skewness	-0.668	-0.344	-0.289
Standard Deviation	0.128	0.174	0.180
Number of countries	188	188	188
Proportion < 0.3	0.000	0.027	0.022
Proportion ≥ 0.3 & ≤ 0.7	0.250	0.553	0.468
Proportion > 0.7	0.750	0.420	0.510

Table A.2. Top 20 country rankings with the most optimistic weighting scheme of HDI in 2015

Country	Health	Education	GNI	HDI - G	HDI - O	Rank Geo.	Rank Opt.	Δ Rank
Hong Kong	0.987	0.822	0.951	0.917	0.983	12	1	11
Singapore	0.972	0.814	1.000	0.925	0.975	5	2	3
Japan	0.980	0.842	0.894	0.904	0.971	17	3	14
Switzerland	0.971	0.891	0.957	0.939	0.970	2	4	-2
Italy	0.974	0.814	0.879	0.887	0.964	26	5	21
Iceland	0.965	0.906	0.894	0.921	0.958	9	6	3
Australia	0.962	0.939	0.915	0.938	0.957	3	7	-4
Spain	0.966	0.818	0.875	0.884	0.957	27	8	19
Sweden	0.959	0.855	0.927	0.913	0.956	14	9	5
Luxembourg	0.952	0.783	0.972	0.898	0.954	20	10	10
France	0.959	0.839	0.898	0.897	0.953	21	11	10
Canada	0.957	0.890	0.915	0.920	0.953	10	12	-2
Norway	0.949	0.916	0.984	0.949	0.953	1	13	-12
Israel	0.962	0.870	0.868	0.899	0.952	19	14	5
Korea, Rep.	0.956	0.867	0.883	0.901	0.948	18	15	3
Netherlands	0.949	0.897	0.927	0.924	0.947	7	16	-9
New Zealand	0.954	0.917	0.875	0.915	0.946	13	17	-4
Andorra	0.946	0.718	0.933	0.859	0.945	32	18	14
Austria	0.947	0.820	0.918	0.893	0.944	24	19	5
Chile	0.953	0.784	0.812	0.847	0.938	39	20	19

Notes: Health, Education, GNI columns present the respective index outcomes.
HDI – G and HDI – O represent HDI values obtained with the geometric mean aggregation and when optimistic weights are used to aggregate sub-indices, respectively.
Rank Geo. and Rank Opt. present the rank positions of countries when geometric mean and optimistic weights are used to obtain HDI outcomes.
Δ Rank represents the difference between the rankings obtained when optimistic weights and geometric mean are used to obtain HDI values where positive (negative) values suggest that country is ranked higher (lower) when the optimistic weights are used compared to geometric mean aggregation case.

Table A.3. Major rank reversals with optimistic case compared to the geometric mean

Panel A. Countries that moved to a higher ranking with the optimistic case

Country	Health	Education	GNI	Rank Geo.	Rank Opt.	Δ Rank
Maldives	0.876	0.561	0.701	105	54	51
Dominica	0.890	0.618	0.697	96	49	47
Vietnam	0.861	0.617	0.601	115	74	41
Lebanon	0.916	0.656	0.739	76	38	38
Nicaragua	0.849	0.542	0.583	124	88	36
Morocco	0.836	0.503	0.646	123	91	32
Honduras	0.821	0.518	0.574	130	100	30
China	0.861	0.631	0.739	90	60	30
Costa Rica	0.917	0.684	0.747	66	36	30
Cuba	0.917	0.779	0.651	68	39	29
Eritrea	0.680	0.267	0.408	180	152	28
Bangladesh	0.800	0.457	0.530	141	113	28
Mexico	0.877	0.655	0.770	77	50	27
Albania	0.892	0.715	0.699	75	48	27
Jamaica	0.859	0.678	0.668	94	68	26
Niger	0.645	0.206	0.330	187	162	25
Ethiopia	0.686	0.318	0.411	174	149	25
Cape Verde	0.824	0.534	0.620	122	97	25
Ecuador	0.863	0.665	0.704	89	64	25

Panel B. Countries that moved to a lower ranking with the optimistic case

Country	Health	Education	GNI	Rank Geo.	Rank Opt.	Δ Rank
Russian Federation	0.773	0.816	0.823	49	110	-61
Kazakhstan	0.763	0.805	0.815	56	115	-59
Belarus	0.792	0.834	0.763	52	104	-52
South Africa	0.579	0.705	0.724	119	168	-49
Lithuania	0.823	0.882	0.840	37	86	-49
Trinidad and Tobago	0.777	0.717	0.851	65	107	-42
Swaziland	0.445	0.545	0.653	148	187	-39
Botswana	0.685	0.658	0.753	108	143	-35
Palau	0.813	0.808	0.744	61	94	-33
Angola	0.503	0.482	0.626	150	182	-32
Azerbaijan	0.783	0.723	0.770	78	108	-30
Nigeria	0.509	0.477	0.604	152	181	-29
Equatorial Guinea	0.583	0.439	0.811	136	164	-28
Gabon	0.691	0.618	0.793	109	137	-28
Ukraine	0.787	0.803	0.649	84	112	-28
Latvia	0.836	0.835	0.819	44	72	-28
Lesotho	0.463	0.503	0.529	159	186	-27
Seychelles	0.820	0.706	0.827	63	89	-26

Notes: Health, Education, GNI columns present the respective index outcomes. Rank Geo. and Rank Opt. present the rank positions of countries when geometric mean and optimistic weights are used to obtain HDI outcomes. Δ Rank represents the difference between the rankings obtained when optimistic weights and geometric mean are used to obtain HDI values where positive (negative) values suggest that country is ranked higher (lower) when the optimistic weights are used compared to geometric mean aggregation case.

Table A.4. Ranking of countries with the optimistic weighting scheme and geometric mean aggregation in 2015								
Country	(O-G)	Δ	Country	(O-G)	Δ	Country	(O-G)	Δ
Hong Kong	1-12	11	Poland	41-36	-5	Bulgaria	81-57	-24
Singapore	2-5	3	Panama	42-60	18	Venezuela	82-72	-10
Japan	3-17	14	Oman	43-53	10	Saint Kitts & Nevis	83-74	-9
Switzerland	4-2	-2	Croatia	44-45	1	Georgia	84-70	-14
Italy	5-26	21	Bahrain	45-47	2	Armenia	85-85	0
Iceland	6-9	3	Estonia	46-30	-16	Lithuania	86-37	-49
Australia	7-3	-4	Uruguay	47-54	7	Colombia	87-95	8
Spain	8-27	19	Albania	48-75	27	Nicaragua	88-124	36
Sweden	9-14	5	Dominica	49-96	47	Seychelles	89-63	-26
Luxembourg	10-20	10	Mexico	50-77	27	Jordan	90-86	-4
France	11-21	10	Slovakia	51-40	-11	Morocco	91-123	32
Canada	12-10	-2	Argentina	52-46	-6	Dominican Rep.	92-100	8
Norway	13-1	-12	Ant. & Barbuda	53-62	9	Grenada	93-80	-13
Israel	14-19	5	Maldives	54-105	51	Palau	94-61	-33
Korea, Rep.	15-18	3	Montenegro	55-48	-7	St. Vincent & Grenadines	95-99	4
Netherlands	16-7	-9	Kuwait	56-51	-5	El Salvador	96-117	21
New Zealand	17-13	-4	Bosnia.	57-81	24	Cape Verde	97-122	25
Andorra	18-32	14	Bahamas	58-58	0	Samoa	98-104	6
Austria	19-24	5	Saudi Arabia	59-38	-21	Paraguay	99-110	11
Chile	20-39	19	China	60-90	30	Honduras	100-130	30
Germany	21-4	-17	Hungary	61-43	-18	Palestine	101-114	13
Ireland	22-8	-14	Barbados	62-55	-7	Tonga	102-101	-1
Belgium	23-22	-1	Turkey	63-71	8	Libya	103-102	-1
Finland	24-23	-1	Ecuador	64-89	25	Belarus	104-52	-52
Liechtenstein	25-15	-10	Iran	65-69	4	Suriname	105-97	-8
United Kingdom	26-16	-10	Malaysia	66-59	-7	Guatemala	106-126	20
Portugal	27-41	14	Macedonia	67-82	15	Trinidad & Tobago	107-65	-42
Greece	28-29	1	Jamaica	68-94	26	Azerbaijan	108-78	-30
Denmark	29-6	-23	Romania	69-50	-19	Egypt	109-111	2
Malta	30-33	3	Algeria	70-83	13	Russia	110-49	-61
Slovenia	31-25	-6	Serbia	71-67	-4	Moldova	111-107	-4
Cyprus	32-34	2	Latvia	72-44	-28	Ukraine	112-84	-28
Brunei	33-31	-2	Mauritius	73-64	-9	Bangladesh	113-141	28
United States	34-11	-23	Vietnam	74-115	41	Vanuatu	114-134	20
Qatar	35-35	0	Saint Lucia	75-92	17	Kazakhstan	115-56	-59
Costa Rica	36-66	30	Sri Lanka	76-73	-3	Fiji	116-91	-25
Czech Republic	37-28	-9	Brazil	77-79	2	Mongolia	117-93	-24
Lebanon	38-76	38	Thailand	78-87	9	Iraq	118-121	3
Cuba	39-68	29	Tunisia	79-98	19	Belize	119-103	-16
UAE	40-42	2	Peru	80-88	8	Bhutan	120-132	12

Table A.4. Continues...

Country	(O-G)	Δ	Country	(O-G)	Δ	Country	(O-G)	Δ
Kyrgyzstan	121-120	-1	Kiribati	144-137	-7	Benin	167-167	0
Indonesia	122-113	-9	Tanzania	145-151	6	South Africa	168-119	-49
Uzbekistan	123-106	-17	Madagascar	146-158	12	Togo	169-166	-3
Nepal	124-144	20	Sudan	147-165	18	Uganda	170-163	-7
Bolivia	125-118	-7	Rwanda	148-160	12	Zimbabwe	171-155	-16
Philippines	126-116	-10	Ethiopia	149-174	25	Burkina Faso	172-185	13
Tajikistan	127-129	2	Yemen	150-168	18	Mali	173-175	2
Syria	128-149	21	Congo	151-135	-16	Guinea	174-183	9
Micronesia	129-127	-2	Eritrea	152-180	28	Congo, Dem. Rep.	175-176	1
Timor-Leste	130-133	3	Mauritania	153-157	4	Cameroon	176-153	-23
India	131-131	0	Malawi	154-170	16	South Sudan	177-181	4
Cambodia	132-143	11	Papua N. Guinea	155-154	-1	Burundi	178-184	6
Turkmenistan	133-112	-21	Comoros	156-161	5	Guinea-Bissau	179-178	-1
Guyana	134-128	-6	Haiti	157-164	7	Mozambique	180-182	2
Solomon Islands	135-156	21	Djibouti	158-172	14	Nigeria	181-152	-29
Laos	136-138	2	Kenya	159-146	-13	Angola	182-150	-32
Gabon	137-109	-28	Ghana	160-139	-21	Côte d'Ivoire	183-171	-12
Pakistan	138-147	9	Zambia	161-140	-21	Chad	184-186	2
Myanmar	139-145	6	Niger	162-187	25	Sierra Leone	185-179	-6
Sao Tome & Prin.	140-142	2	Afghanistan	163-169	6	Lesotho	186-159	-27
Senegal	141-162	21	Equatorial Guinea	164-136	-28	Swaziland	187-148	-39
Namibia	142-125	-17	Gambia	165-173	8	Central African Rep.	188-188	0
Botswana	143-108	-35	Liberia	166-177	11			

Notes: (O-G) represents the ranking of the country when optimistic weights and geometric mean are used to aggregate HDI values. Δ represents the difference between the rankings obtained with optimistic weights and geometric mean are used to obtain HDI values where positive (negative) values suggest that country is ranked higher (lower) when the optimistic weights are used compared to geometric mean aggregation case.