Numerical Modelling of Aerodynamics for Applications in Sports Car Engineering

Dr. Pascual Marqués-Bruna, Faculty of Arts & Sciences, Edge Hill University,
St. Helen’s Road, Ormskirk, L39 4QP, United Kingdom. marquesp@edgehill.ac.uk

Elena Spiridon, School of Natural Sciences and Psychology, Liverpool John Moores University,
Tom Reilly Building, Byrom Street, Liverpool, L3 3AF, United Kingdom. E.Spiridon@2008.ljmu.ac.uk

Abstract

This paper demonstrates the effectiveness of numerical modelling of aerodynamics as a problem-based-learning strategy for the implementation of aerodynamics theory in sports car engineering programs. Examples are presented of multidisciplinary projects that apply fundamental aerodynamics theory (*Thin-airfoil Theory*, *Vortex Panel Method*, *Finite-wing Theory* and *Lifting-line Theory*), mathematical models and computer aided design in the conceptual design of inverted wings for the Grand Touring sports car. It is suggested that a numerical modelling problem-based-learning project commences with an analysis of airfoil geometry and mean camber line followed by simulation of surface velocity, pressure distribution and boundary layer development at low Reynolds numbers. Airfoil properties are subsequently used to inform the construction of the three-dimensional wing; where, wing planform, taper and flow control devices including a Gurney flap and end plates must be adjusted to enhance aerodynamic efficiency and comply with motor racing regulations. Optimised geometric or aerodynamic twist may be implemented for the twisted wing to replicate the high performance of a straight wing of elliptic planform and yield high downforce and minimum possible induced drag. Validation tests ultimately determine the accuracy of a computational method in predicting wing performance and should be an integral component of any problem-based-learning project. Numerical modelling of aerodynamics is suggested as a problem-based-learning strategy for Engineering students to undertake complex multi-disciplinary real-world projects that promote initiative and creativity.

Keywords: Aerodynamics, Computer simulation, Engineering education, Sports car wing.

Introduction

Problem-based-learning (PBL) is a learning strategy in which Engineering students have been observed to work cooperatively and effectively to solve complex open-ended real-world problems **[1,2]**. PBL tasks are typically multi-disciplinary in nature, involve the integration of theoretical concepts and rely on students’ initiative and creativity. For Engineering students, it is important that course projects integrate the hands-on experiential work characteristic of PBLs **[2,3]**. Numerical modelling of aerodynamics represents a valuable PBL strategy in sports car engineering programs that permits a comprehensive exploration of aerodynamics theory and the seamless integration of several disciplines; namely, Aeronautical Engineering, Mathematics, computer aided design (CAD) and computer aided engineering (CAE). Fundamental aerodynamics theories that lend themselves to numerical modelling include the *Thin-airfoil Theory*, *Vortex Panel Method*, *Finite-wing Theory* and *Lifting-line Theory* **[4,5,6]**.

Preliminary design of the wing for a Grand Touring (GT) sports car usually entails an evaluation of airfoil geometry and aerodynamic properties including mean camber line, surface flow velocity and pressure distribution. The effects of viscosity and boundary layer development and stability (thickness, transition and separation) are also investigated and airfoil coefficients of lift (c_l), drag (c_d) and pitch are calculated **[4,7]**. The effects of Reynolds number (Re) upon airfoil and wing aerodynamics are prominent. Unlike aircraft wings, the wing of a GT sports car operates at low *Re* and any airfoil originally designed for aviation will be functioning in off-design conditions **[5,8]**. The next stage in the analysis typically addresses the design of the three-dimensional wing, where it is important to account for wing dimensions to comply with motor racing regulations and adjust wing aerodynamics according to the characteristics of the racing circuit **[8]**. Wing design is usually assisted by CAD and CAE and requires a good understanding of wing geometry, including planform, taper, aspect ratio (AR) and Oswald efficiency factor (e). Wing construction necessitates also an appreciation of three-dimensional flow, tip vortex and downwash configuration and their effect upon induced drag. Flow control and high-lift devices consisting of vortex generators, a Gurney flap, winglets and end plates are subsequently applied **[5,9,10]**. Numerical modelling has been used in past research to optimise wing efficiency and investigate modern concepts in adaptive wing technology **[11]**. The work of Phillips, Phillips *et al.* and Phillips and Alley **[12-14]** explains the computation of spanwise wing twist and the use of full-span twisterons to reduce induced drag. The twist may be geometric, aerodynamic or a combination of both **[6,15]**. Thus, there is a logical sequence in wing design from airfoil analysis through to wing twist optimisation.

Validation of computed aerodynamic coefficients is an essential element of numerical analysis that ultimately determines whether the aerodynamic properties of a new wing design can be predicted with accuracy. Thus, calculated aerodynamic values are typically compared to experimental data obtained using a wind tunnel **[e.g., 7,13]**. Validation is not only limited to aerodynamic coefficients but it can be used to assess the accuracy of CAE and computational fluid dynamics (CFD) software in predicting boundary layer behaviour. This paper demonstrates the effectiveness of numerical modelling of aerodynamics as a PBL strategy for the implementation of aerodynamics concepts in sports car engineering programs.

Numerical modelling of aerodynamics

The work of Marqués-Bruna **[7]** provides an example of the application of the *Vortex Panel Method* **[6]** to examine the adaptation of aircraft airfoils for sports cars. Different airfoils developed by the National Advisory Committee for Aeronautics (NACA) and airfoils with extensive natural laminar flow (NLF) are examined. Airfoil ordinates and mean camber line equations are obtained from the literature and the airfoil geometry is reconstructed using CAD. For example, the mean camber line for the NACA 65_1-412 airfoil is defined mathematically by eqs. (1) and (2)

$$\frac{z}{c} = -\frac{c_{l0}}{4\pi}\left[\left(1-\frac{x}{c}\right)\ln\left(1-\frac{x}{c}\right)+\frac{x}{c}\ln\left(\frac{x}{c}\right)\right] \tag{1}$$

and

$$\frac{dz}{dx} = \frac{c_{l0}}{4\pi}\left[\ln\left(1-\frac{x}{c}\right)-\ln\left(\frac{x}{c}\right)\right] \tag{2}$$

where, z and x are Cartesian coordinates, c is the airfoil chord and c_{l0} is the design lift coefficient of the airfoil **[4]**. Fig. 1 illustrates the principles of the *Vortex Panel Method* for the computation of lift using eq. (3) **[6]**.

$$V_\infty cos\beta_i - \sum_{j=1}^{n}\frac{\gamma_j}{2\pi}\int_j \frac{\partial\theta_{ij}}{\partial n_i}ds_j = 0 \tag{3}$$

where, V_∞ is the freestream velocity, β_i is the angle between V_∞ and n_i, i is the control point at which the vortex strength is being calculated, n is the panel number, j is the panel which is inducing some vortex at i, γ_j is the vortex strength at j, n_i is the unit vector normal to the ith panel, θ_{ij} is the angle between panels i and j, and s_j is the length of panel j. In Fig. 1, x and y are the coordinates of the control points at panels i and j. Unlike *Thin-airfoil Theory*, the *Vortex Panel Method* can be used for airfoils of thickness greater than 12% **[5,6]**. However, the *Vortex Panel Method* assumes an inviscid flow and drag coefficients must be obtained using other computational methods, such as boundary layer equations for low-speed incompressible laminar and turbulent flow **[4,5]**.

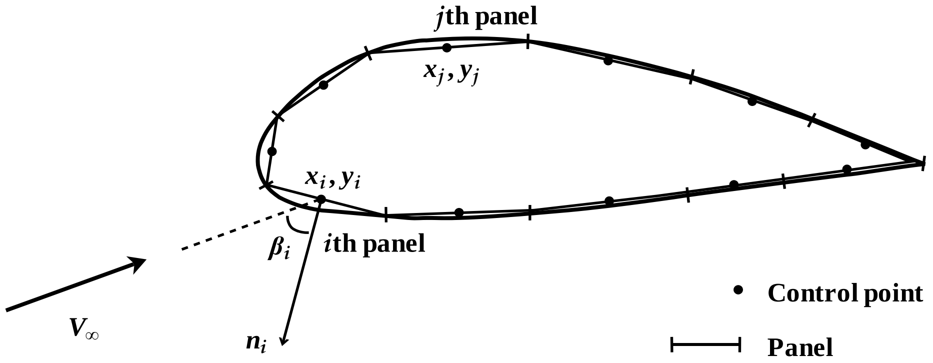

Fig. 1: Reconstruction of an airfoil surface using a series of vortex panels (adapted from [7], with permission).

Validation tests of AeroFoil 2.2 software were conducted by Marqués-Bruna **[7]** using drag polars and airfoils of various geometries set angles of attack (α) of -4° ≤ α ≤ 12° and operating at Re = 2.0, 3.0 and 3.1 × 10^6. Computed and published wind tunnel experimental data **[e.g., 4]** were in close agreement (Fig. 2).

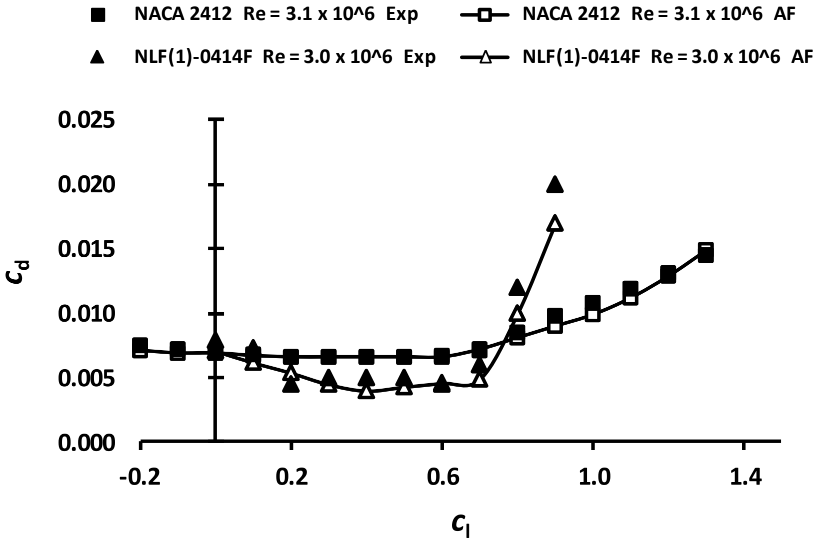

Fig. 2: Experimental wind tunnel data (Exp) and calculated values obtained using AeroFoil 2.2 software (AF) for two airfoils (adapted from [7], with permission).

Drag polars help the Engineering student observe clearly the non-linear development of drag as lift increases linearly. For illustrative purposes, sample theoretical drag polars for two airfoils operating at Re = 1.29 × 10^6 are shown in Fig. 3 **[7]**. The NACA 65_1-412 stalls abruptly at c_l of around 1.3, which corresponds to α = 9°. The NLF(1)-0414F attains low drag in the range of c_l within its low drag bucket and shows stall resistance, which are desirable features for applications in sports car design. However, the two airfoils, originally designed for aircraft, yield decreased aerodynamic efficiency at low Re; characterised by less lift, higher drag and a narrower low-drag bucket. Hence, numerical modelling shows that the aerodynamic performance of airfoils deteriorates when operating in off-design conditions **[4,6]**.

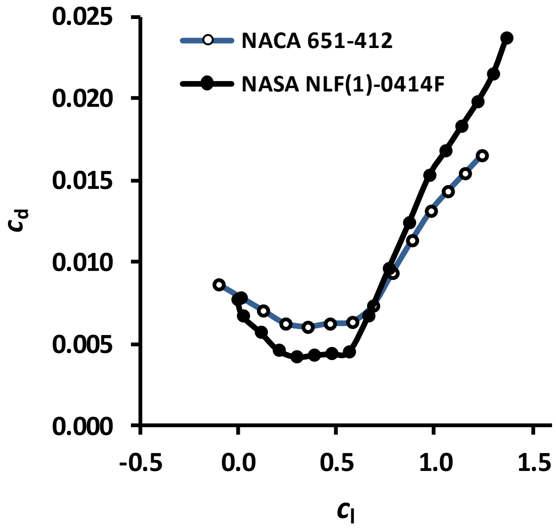

Fig. 3: Drag polars at Re = 1.29 × 10^6
(adapted from [7], with permission)

An application of *Finite-wing Theory* **[4,5]** in the design of the wing for a GT sports car is found in the work of Marqués-Bruna and Grimshaw **[10]**. Wing dimensions are adjusted to optimise AR and comply with car design regulations **[8]**; Fig. 4. Wing taper is introduced to correct for planform deviations from a planform with an elliptical lift distribution and to enhance Oswald efficiency **[6]**.

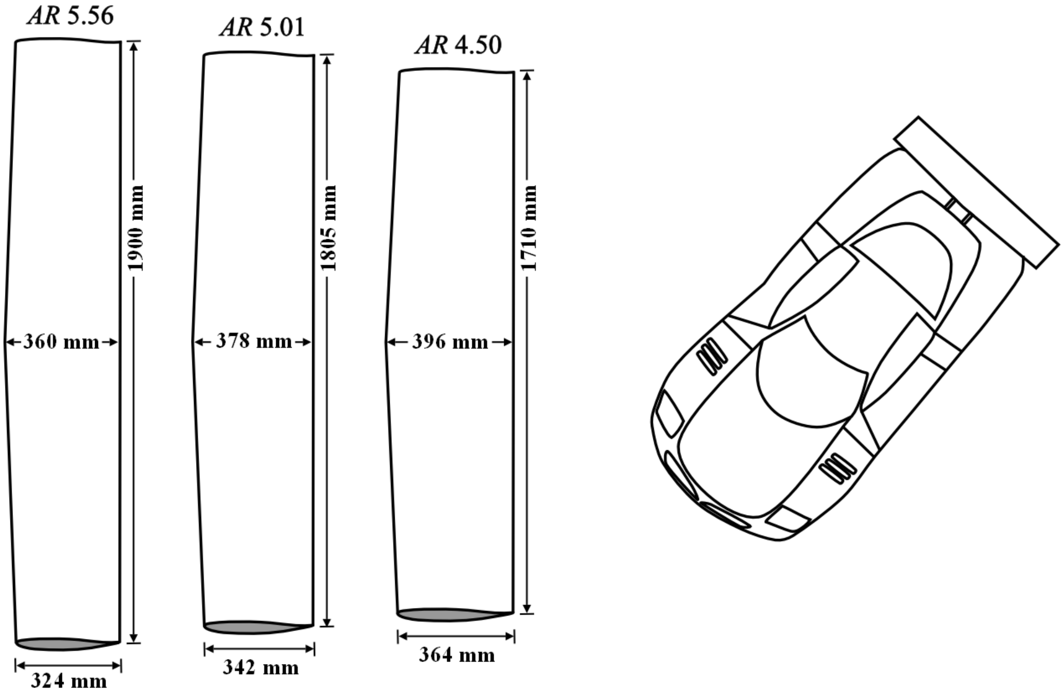

Fig. 4: Dimensions and AR for three wings of planform area = 0.65 m^2
for a GT sports car (adapted from [10], with permission).

The wing lift slope (a) is obtained from eq. (4)

$$a = \frac{a_0}{1 + \left(\frac{a_0}{\pi A\mathrm{R}}\right)\,(1 + \tau)} \tag{4}$$

where, a_0 is the airfoil lift slope and τ is the induced lift efficiency factor obtained from graphical data **[e.g., 4,6]**. Because the wing is inverted in a sports car, the wing downforce coefficient $(-C_L)$, rather than the lift coefficient, is calculated. The $-C_L$ is obtained using eq. (5)

$$-C_L = a(\alpha - \alpha_{\mathrm{L0}}) \tag{5}$$

where, α_{L0} is the zero-lift angle of attack. The expression for wing profile drag coefficient (C_D) is eq. (6)

$$C_D = c_{di} + \frac{(-C_L)^2}{\pi e A\mathrm{R}} \tag{6}$$

In eq. (6), c_{di} is the c_d adjusted for induced flow which is obtained from graphical experimental data **[e.g., 4]** using the c_l adjusted for induced flow (c_{li}) sensed by the airfoil. Using numerical modelling, Marqués-Bruna and Grimshaw **[10]** evaluated the lift-to-drag ratio $(-C_L/C_D)$ at low Re of NACA and low speed (LS) wings of different AR (Fig. 5). In Fig. 5, the data refer to baseline wings, hence devoid of any flow control and high-lift devices.

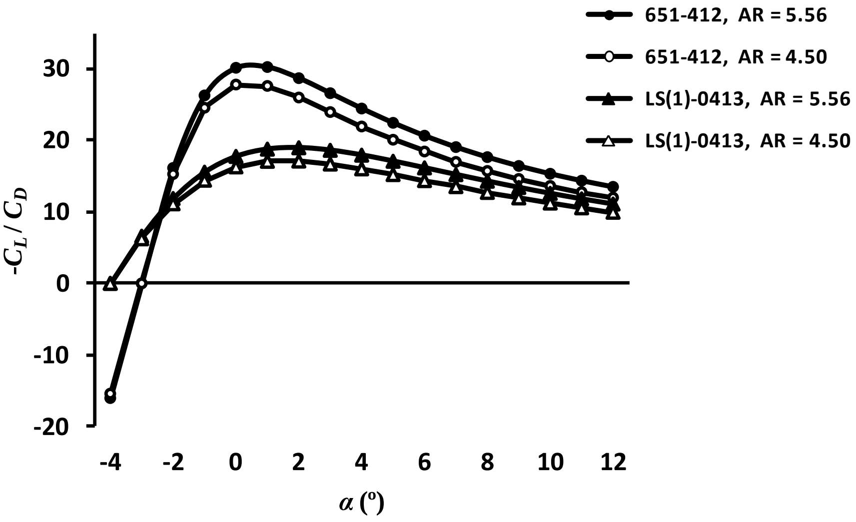

Fig. 5: Wing $-C_L/C_D$ according to constituent airfoil and *AR*
(from [10], with permission)

Optimisation of the sports car wing using spanwise twist can be carried out using numerical modelling. Wing twist is an effective method of minimising induced drag **[5,11]**. Phillips, Phillips *et al.* and Phillips and Alley **[12-14]** developed a modified version of the classical *Lifting-line Theory* for the implementation of geometric and aerodynamic twist (*washout*) to enhance the aerodynamic efficiency of aircraft wings. Optimised geometric twist of a wing for a GT sports car was examined by Marqués-Bruna **[15]**. Fig. 6 depicts the inverted wing fitted with end plates and a Gurney flap; where, h_{ep} is the height of the end plate and h_G is the height of the Gurney flap. The optimised twist was applied by finding the optimum total twist (Ω_{opt}) and applying a normalised elliptical twist distribution ($\omega(\theta)_{opt}$) along the span (b); based on Phillips **[12]** and Phillips *et al.* **[13]**.

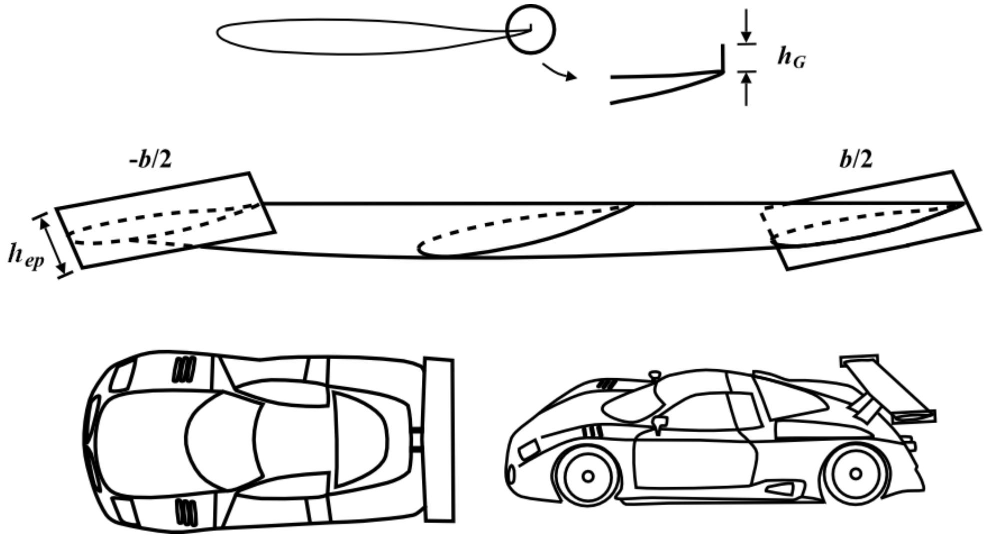

Fig. 6. A CAD-generated elliptically-twisted wing fitted with a Gurney flap and end plates installed on a GT sports car (adapted from [15], with permission).

The normalised $\omega(\theta)_{opt}$ is given by the function

$$\omega(\theta)_{opt} = 1 - \frac{sin(\theta)}{1 - (1 - R_T)|cos\theta|} \tag{7}$$

where, θ represents the spanwise angular coordinate and Ω_{opt} is given by

$$\Omega_{opt} = \frac{k_{DL} c_{l0}}{2 k_{D\Omega}} \times \frac{1}{a} \tag{8}$$

In eq. (8), k_{DL} is the lift-twist factor and $k_{D\Omega}$ is the twist factor. The induced drag coefficient (C_{Di}) for optimally-twisted wings is calculated using eq. (9) **[12]**.

$$C_{Di} = \frac{-C_L^2}{\pi AR} + \frac{\delta}{\pi AR}\left[-C_L - \frac{\pi a_0 \Omega}{2(1 + R_T)}\right]^2 \tag{9}$$

where, δ is the induced drag factor, Ω is the pre-set approximated total wing twist and R_T is the taper ratio. The c_l and AR are adjusted to account for the increased downforce generated by the Gurney flap **[9]** and the flow control and increased effective AR achieved by using end plates in the GT car **[8]**, respectively, using eqs. (10) and (11).

$$\Delta c_l = \sqrt{\frac{h_G}{c}} \tag{10}$$

and

$$AR_{end\,plate} = AR_{actual}\left[1 + 1.9\left(\frac{h_{ep}}{b}\right)\right] \tag{11}$$

where, Δc_l is the increment in c_l caused by the Gurney flap, c is the airfoil chord and $AR_{end\,plate}$ is the effective AR that results from using end plates. A comparative analysis of straight and optimally twisted wings, fitted with a Gurney flap and end plates, is shown in Fig. 7. For twisted wings α refers to α at the wing tips. The C_{Di} is larger in the twisted wings, however it must be worn in mind that the geometric twist increases α over the mid span region by Ω_{opt} = 4.6° in the NACA 65_1-412 wing and Ω_{opt} = 4° in the LS(1)-0413 wing. The increased α augments C_L, C_D and C_{Di}.

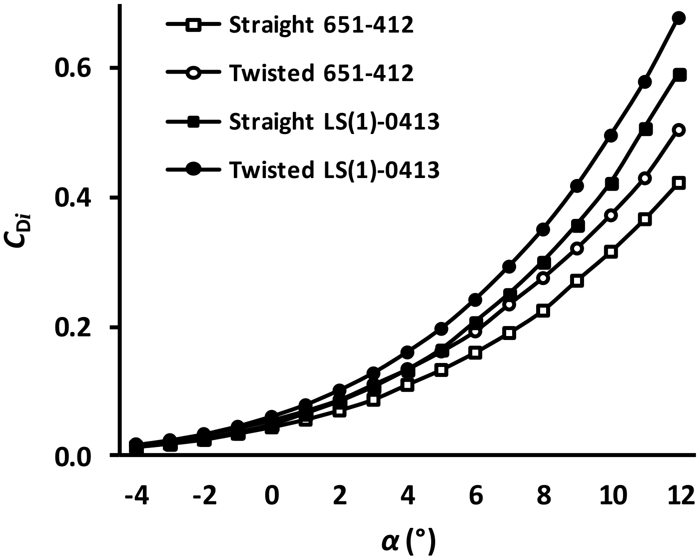

Fig. 7. C_{Di} for straight and elliptically-twisted wings (adapted from [15], with permission).

Discussion

This paper demonstrates the effectiveness of numerical modelling of aerodynamics as a PBL project in facilitating the seamless integration of aerodynamics theory and laboratory practice for the improvement of the Engineering student's analysis, design and troubleshooting capabilities **[1,2]**. The validation of the *Vortex Panel Method* carried out by Marqués-Bruna **[7]** suggests that the accuracy of the numerical method is acceptable for the analysis of airfoil aerodynamics (Fig. 2). Nonetheless, accuracy was lower for the NLF(1)-0414 airfoil probably due to the complex geometry and subtle viscous effects in this airfoil. Therefore, validity is airfoil-type dependent and Engineering students should conduct their own validity tests when using airfoils of complex geometry. The validity of the modified *Lifting–line* numerical solution for the prediction of aerodynamic coefficients for optimally-twisted wings was evaluated by Phillips *et al*. **[13]** using CFD. The modified lifting-line method slightly underestimates the decrease in $C_{\mathrm{D}i}$ achieved by optimum twist. Thus, the GT car wing is expected to generate less induced drag than that predicted by validated numerical modelling.

The *Vortex Panel Method* permits the computation of lift, since vortices have circulation, in conditions of incompressible flow and the method is therefore suitable for applications in sports car engineering **[5,8]**. Also, the restriction of *Thin-airfoil Theory* to geometric α below the stall **[5,6]** is eliminated. The *Vortex Panel Method* has been supplemented with boundary layer equations in the work of Marqués-Bruna **[7]** for the computation of drag coefficients. Engineering students may also use *Thin-airfoil Theory* in the design of thin wings set at low incidence intended for fast motor racing circuits **[8]**. *Thin-airfoil Theory* was developed by Prandtl during World War I **[6]**. The theory assumes an inviscid irrotational flow, since at low incidence the boundary layer remains thin and attached to the airfoil surface. In airfoils of thickness less than 12% of chord length the airfoil is represented by a vortex sheet along its mean camber line to make the airfoil a streamline of the flow. Airfoil aerodynamics are obtained by applying the circulation theory of lift to the streamline **[3,5]**. Thus, using numerical modelling, Engineering students can explore the theoretical constructs that underpin *Thin-airfoil Theory* and the *Vortex Panel Method* for the design of the GT sports car wing.

Airfoils originally designed for aviation are less efficient at low *Re* **[5,7]**. However, the Engineering student should consider that the NACA 65_1-412 and the NLF(1)-0414F airfoils show considerable aerodynamic efficiency at low Re for the range of c_l within their low-drag bucket (Fig. 3). Drag increments at high α in the NACA 65_1-412 can be attributed to its sharp leading edge that triggers early transition, and rapid turbulent boundary layer thickening and separation **[4]**. The small-radius leading edge causes the NACA 65_1-412 airfoil to stall at high α, thus this airfoil may be used at low incidence as a stabiliser in fast racing circuits **[7,8]**. In the NLF(1)-0414F, the steep increase in c_d beyond the upper boundary of the low-drag bucket at low Re (Fig. 3) has been attributed to its thick profile (14%), moderate camber (2.70%) and steep aft pressure recovery **[7]**. An interesting observation is that the NLF(1)-0414F airfoil is stall resistant at high incidence due to a thicker leading edge than typical for NLF airfoils **[6]**. The numerical modelling unveils the versatility of the NLF(1)-0414F airfoil for sports cars due to its wide low-drag bucket, low minimum c_d and stall resistance.

Numerical modelling allows an evaluation of wing aerodynamic efficiency. Analysis based upon *Lifting-line Theory* indicates an advantage of applying taper to a rectangular wing of low AR for the sports car. Due to taper, the planform of the wing of AR = 4.50 more closely resembles an elliptical planform (Fig. 4) and the wing achieves near-unity Oswald efficiency **[10]**. Aerodynamic efficiency attained by operation at the best $-C_L/C_D$ is essential in aircraft **[5]**. However, in a race car, a wing incidence for peak $-C_L/C_D$ does not result in high levels of downforce. For example, the NACA 65_1-412 wing obtains the best $-C_L/C_D$ at α = 0° (Fig. 5), however $-C_L$ is only 0.2. At α = 4°, $-C_L/C_D$ is below the maximum, however $-C_L$ increases to 0.5. Thus, the wing may be set at an incidence that gives high downforce at the expense of some extra profile drag. An LS(1)-0413 airfoil yields superior downforce due to its large leading edge radius, extended favourable pressure gradient and rapid concave pressure recovery **[6]**. However, the LS(1)-0413 wing shows low maximum $-C_L/C_D$ (Fig. 5) due to its blunt leading edge and profile thickness of 13% that contribute to form drag. Thus, a PBL project should incorporate an evaluation of Oswald efficiency and $-C_L/C_D$.

The numerical modelling of Marqués-Bruna **[15]** suggests that a twist-optimised wing of linearly-tapered planform for the GT car nearly duplicates the high aerodynamic performance of a wing of elliptic planform and helps achieve minimum possible $C_{\mathrm{D}i}$, consonant with theory **[5,14]**. The larger leading edge radius and 1% greater thickness of the LS(1)-0413 airfoil, compared to the NACA 651-412 airfoil, augment $C_{\mathrm{D}i}$ **[4]**. However, the increased $C_{\mathrm{D}i}$ in the twisted wings can be solely attributed to the increased mid-span incidence associated with twisting the wing (Fig. 7). The findings of Marqués-Bruna **[15]** suggest that stall control may be achieved using a stall resistant LS(1)-0413 airfoil, geometric twist and a small R_T. Geometric twist can generate a partial, rather than complete, stall and a small R_T helps preserve chord length and prevent a low Re and the stall in the wing tip region **[4]**. Numerical modelling can be used to tailor wing design according to the characteristics of different racing circuits. Thus, the wing for a GT car may be constructed with high downforce or with high aerodynamic efficiency in mind **[8,9]**. A twisted LS(1)-0413 wing generates greater downforce. However, higher efficiency is achieved using a straight NACA 65_1-412 wing due to the small-radius leading edge and 1% thinner profile of the constituent airfoil **[4]**. End plates and Gurney flaps provide modified wake configuration, flow control and increased downforce **[9]**. Interestingly, end plates reduce Oswald efficiency but augment AR which enhances wing efficiency **[5]**. The Gurney flap is a mechanically simple high-downforce system that increases the effective camber of the airfoil and produces sizeable increments in both lift and drag. A separation bubble immediately upstream of the flap deflects the streamlines downwards (upwards in an inverted airfoil) and enhances lift. The flap augments the loading along the entire airfoil, but particularly at the suction peak and near the trailing edge. The optimum height for a Gurney flap is h_G = 2% of c **[9]** and the flap must be of the order of the boundary layer thickness. However, the Gurney flap in a GT car if of h_G = 15 mm or higher, which causes drag **[8]**. Thus, the Engineering student can use numerical modelling to explore the effects of wing twist, end plates and a Gurney flap prior to the construction of a prototype wing for the GT car.

The research presented in this paper identifies potential future projects that may be undertaken by Engineering students. The numerical analysis of Marqués-Bruna and Grimshaw **[10]** was restricted to conditions of smooth airfoil surface. In future work, the assessment may extend to the introduction of surface roughness and thus early boundary layer transition at low Re. In an actual GT race, transition may occur earlier due to aerodynamic roughness caused by fabrication methods, dust and rain droplets **[8]**. Also, the effect of vortex generators on boundary layer stability at low Re needs investigating in future work. CFD analysis may help visualize the flow field in which the end plates operate and adjust the tip device to give desired performance.

Conclusion

This paper demonstrates the effectiveness of numerical modelling of aerodynamics in the application of aerodynamics theory in the design of the GT sports car wing. The validation of the modified *Lifting–line* numerical solution for optimally twisted wings **[13]** and the *Vortex Panel Method* **[7]** suggests that numerical modelling predicts aerodynamic properties with acceptable accuracy. Using numerical modelling, the Engineering student can design complex optimally-twisted wings in which the twist varies elliptically along b. The twist-optimised wing of linearly-tapered planform for the GT car should replicate the high aerodynamic performance of a wing of elliptic planform and yield high downforce and minimum possible $C_{\mathrm{D}i}$. End plates enhance the effective AR and integration of a Gurney flap produces generous increments in $-C_L$ through a mechanically simple device that augments the effective camber of the airfoil. Numerical modelling of aerodynamic is a constructive multi-disciplinary PBL strategy that permits an exploration of aerodynamics theory (e.g., *Thin-airfoil Theory*, *Vortex Panel Method*, *Finite-wing Theory*, *Lifting-line Theory*) and principles of wing operation at low Re. In addition, the use of numerical modelling for applications in real-world sports car engineering promotes group work, decision-making, initiative and creativity.

Acknowledgement

The authors would like to acknowledge the financial support provided by Edge Hill University for this project.

Authors' biographies

Dr Pascual Marqués obtained his MPhil from Brunel University in 1998 and his PhD from Liverpool John Moores University in 2005, both in Biomechanics. Dr Marqués has been a Lecturer at The University of Exeter, Brunel University and South Bank University and he is currently at Edge Hill University in the United Kingdom. His research involves numerical modelling of aerodynamics for applications in Sports Engineering using CAD, MATLAB and AeroFoilEngineering software. Recent projects include the optimization of aerodynamic stability in ski jumping and wing design for sports cars.

Elena Spiridon is a Lecturer in Psychology at Liverpool John Moores University and was formerly a lecturer at Edge Hill University. Elena obtained her BSc (Hons) Psychology (First Class degree classification) from Lancaster University in 2007. She was granted the 2007 GBR British Psychological Society Award for the highest average University mark. Elena is a member of the REFLECT project research team within the 7th Framework Programme (FP7) funded by the European Commission. Her research interest is in the development of a prototype biocybernetic system that measures affective-motivational states via psychophysiology and provides feedback to the user in real-time.

References

1. Perrenet JC, Bouhuijis PA and Smits JG. (2000). The suitability of problem-based learning for Engineering Education: Theory and practice. *Teaching in Higher Education*. ***5***(33): 345-358.
2. Hunt EM, Lockwood-Cooke P and Kelley J. (2010). Linked-class problem-based learning in Engineering: Method and evaluation. *American Journal of Engineering Education*. ***1***(1): 79-87.
3. Ahmed NA. (2005). Demonstration of the effectiveness and limitations of thin airfoil theory in the aerodynamic study of airfoil characteristics. *International Journal of Mechanical Engineering Education*. ***32***(4): 271-282.
4. Abbott IH and von Doenhoff AE. (1959). *Theory of Wing Sections*. Dover Publications, New York.
5. Milne-Thomson LM. (1973). *Theoretical Aerodynamics*. Dover Publications Inc. New York.
6. Anderson JD (2007). *Fundamentals of Aerodynamics*. McGraw-Hill, London.
7. Marqués-Bruna P. (2011). Engineering the race car wing: Application of the vortex panel numerical method. *Journal of Sports Engineering.* ***13***(4): 195-204.

8. Katz J. (2006). *New Directions in Race Car Aerodynamics: Designing for Speed*. Bentley Publishers. Cambridge, Massachusetts.
9. Wang JJ, Li YC and Choi KS. (2008). Gurney flap: Lift enhancement, mechanisms and applications. *Progress in Aerospace Sciences*. ***44***(1): 22-47.
10. Marqués-Bruna P and Grimshaw PN. (2011). Design of the Grand Touring sports car wing. *Proceedings of the Institution of Mechanical Engineers, Part P, Journal of Sports Engineering and Technology.* ***225***(1): 22-31.
11. Stanewsky E. (2001). Adaptive wing and flow control technology. *Progress in Aerospace Sciences*. ***37***(7): 583-667.
12. Phillips WF. (2004). Lifting-line analysis for twisted wings and washout-optimized wings. *Journal of Aircraft*. ***41***(1): 128-136.
13. Phillips WF, Fugal SR and Spall RE. (2006). Minimizing induced drag with wing twist, computational-fluid-dynamics validation. *Journal of Aircraft*. ***43***(2): 437-444.
14. Phillips WF and Alley NR. (2007). Predicting maximum lift coefficient for twisted wings using Lifting-line Theory. *Journal of Aircraft*. ***44***(3): 898-910.
15. Marqués-Bruna P. (2012). Wing design with a twist: Optimised geometric twist of the Grand Touring sports car wing. *Proceedings of the 12th Pan American Congress of Applied Mechanics (PACAM XII)*. *Fluid Mechanics Symposium*. 2nd – 6th January, Port of Spain, Trinidad & Tobago. Pp. 1-6.